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A new application of the Ambartsumian beam theory

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Abstract

The new application of the two degrees of freedom beam theory that takes into account the ability of fibers to resist bending is the main purpose of the present study. In the considered theory, the shape function of Ambartsumian (1958) is involved in the displacement field. Based on the use of the aforementioned displacement functions the strains and stresses are presented where the asymmetric part of the shear stress are provided. Furthermore, the solution of governing equilibrium equations is obtained in which contains terms related to the fiber bending stiffness. Numerical results of the obtained solution are computed and discussed.

Keywords: shape function, fiber bending stiffness, asymmetric part of the shear stress.

1. Introduction

Beam and plate theories have received a numerous attention by many researches. classical beam theory which is known as Euler Bernoulli beam theory was first enunciated circa around 1750 (Bailey, 2013). The rotatory inertia and shear deformation were involved in the first order shear deformation beam theory by Timoshenko in (Timoshenko, 1921). It was pointed out that, Timoshenko investigated the effect of the transverse shear and, the rotatory inertia on the prismatic bars transverse vibration (Ghugal & Sharma, 2011). The well-known Kirchhof assumptions in the plate theory was employed in the theories of Reissner and Stavsky theory, Whitney and Leissa theory and; Ashton theory those established in 1961, 1970 and 1970 respectively (Mantari, Oktem, & Guedes Soares, 2012). It was concluded that the rotary inertia and shear affected significantly on the flexure motions on elastic plates which are isotropic (Mindlin, 1951). Composite laminates in cylindrical bending problem was considered and an exact solution for simply supported plates was presented in (N.J. Pagano, 1969). The study of three elasticity solution of bending of composite laminates that have infinite long and finite size and subjected to sinusoidal loading applied on the its top surface where the exact solution was obtained (N. J. Pagano, 1970). The theory of a refined shear deformation has been studied for flexure of isotropic solid beams that associated with several loading and boundary conditions where the numerical obtained results were compared with their counterparts based on different one-D theory and the exact solution (Ghugal & Sharma, 2011). Theory of hyperbolic shear deformation has been presented for deep fixed-fixed beam where the constitutive relations are used to obtain the transverse shear stresses (Dahake, 2014).

The aforementioned Pagano exact solutions (N.J. Pagano, 1969; N. J. Pagano, 1970) have been used extensively in order to verify the credibility of several one-D and tow-D liner elasticity theory such as (Ali F. Farhat & Soldatos, 15 Apr 2015; Liu, Zhang, & Zhang, 1994; Lü, Lim, & Xu, 2007; Mantari et al., 2012; Kostas P. Soldatos & Watson, 1997a, 1997b; Wang & Shi, 2015). All of that has been in the case where the plates and the beams are reinforced with perfectly flexible fibers. The assumption that the fibers have the ability to resist in bending has been involved theoretically in (Spencer & Soldatos, 2007). Considering the fibers bending stiffness, the linearized, asymmetric 3D elasticity theory has been studied where an additional elastic (fibre bending) modulus is involved in the governing problem equations in addition to 2D plate theory developed with this consideration (Kostas P. Soldatos, 2009). An exact asymmetric 3D linear elasticity solutions for Cylindrical Bending and Vibration of simply supported beams and plates have been presented (Ali. F. Farhat, 2013; Ali F. Farhat & Soldatos, 15 Apr 2015). That exact solutions have been employed to assess the accuracy of various one-D and tow-D plate beam theories such as (A. Farhat & Gwila., May - June 2017; Ali. F. Farhat, 2013; K. P. Soldatos, Aydogdu, M., & Gul, 2019; K. P. Soldatos & Farhat, 20 September 2016). The differences of the displacement field assumed in several beam and plate theories are caused by the number of the degrees of freedom and the shape functions those can determine the transverse shear strains and stresses distribution through the thickness. The shape function of Ambartsumian theory (1958) in addition to some other shape functions which employed different theories were mentioned in (Mantari et al., 2012).

The aim of the present study is to consider the model of polar material plate that presented in (Kostas P. Soldatos, 2009) with the use of the two degrees of freedom displacement which involves the shape function of Ambartsumian (1958). Regarding to these consideration, section 2 formulates the problem where the displacement field will be assumed. Then, the strain and the stress components including the asymmetric stress will be formulated. Consequently, equilibrium equations and the boundary conditions that govern the problem are discussed in section five where the terms those related to the fiber bending stiffness are considered. Furthermore, the Navier-type differential equations system is solved in section six then, numerical results will be presented and discussed in the seventh section. At the end, the significant findings related to the effect of the assumption that the fibers can resist bending will be concluded. Then, recommendation for related future work will be provided.

2. Formulation of the problem

Consider a linearly elastic transversely isotropic homogeneous beam that have thickness h in the z direction, length L in the x direction and one unit in the y direction. The beam middle plane will be assumed to be laying on the Oxy plane. The beam is reinforced with a single family of straight fibers in parallel to x direction that can resist bending. In addition, the beam is assumed to be simply supported on the edges $x = 0$ and $x = L$ and subjected to transverse load $q(x)$ applied on the top beam surface. Furthermore, it would be convenient, as assumed in (K. P. Soldatos & Farhat, 20 September 2016), to assume that the external load applied normally on the top lateral plane of the beam $(z = 1/2)$ has the sinusoidal form as follows:

$$q(x) = q_m \sin(p_m x)\,,\ \ p_m = \frac{m\pi}{L},\ \ m = 1,2, \quad (1)$$

With the use of the shape function of Ambartsumian (1958), the displacement field will be employed in the present paper is in the following form:

$$U(x,z,t) = -z\, w_{,x} + \frac{z}{2}\left[\frac{h^2}{4} - \frac{z^2}{3}\right] u_1(x,t) \quad (2.a)$$

$$W(x,z,t) = w(x,t) \quad (2.b)$$

Where t denotes time and in here as well as the coma denotes the partial differentiation. Furthermore, $U(x,z,t)$ and $W(x,z,t)$ represent displacement functions along x and z directions respectively. Moreover, $w(x,t)$ represents the beam deflection that is assumed to be independent of thickness of the beam.

3. Strains

Inserting the displacement field (2) into the following linear kinematic relations:

$$\boldsymbol{\varepsilon_x} = \frac{\partial U}{\partial x},\ \ \boldsymbol{\gamma_{xz}} = \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x}, \quad (3)$$

Which leads to the following strain field:

$$\boldsymbol{\varepsilon_x} = z\,k_x^c + \frac{z}{2}\left[\frac{h^2}{4} - \frac{z^2}{3}\right] k_x^a, \boldsymbol{\gamma_{xz}} = \left[\frac{h^2}{8} - \frac{z^2}{2}\right] e_{xz}^a \quad , \quad (4)$$

Where,

$k_x^c = -w_{,xx}, k_x^a = u_{1,x}, and\ e_{xz}^a = u_1.$ (5)

Where quantities denoted with a superscript "c" are the same as their classical beam theory counterparts whereas those with a superscript "a" represent the transvers shear deformation effects.

4. Stresses

According to the generalized Hooke's law the normal and the symmetric part of the shear stress are given as shown in (Kostas P. Soldatos, 2009) as follows respectively:

$$\sigma_x = Q_{11}\varepsilon_x,\ \ \tau_{(xz)} = Q_{55}\ \gamma_{xz}, \quad (6)$$

Whereas, the anti-symmetric part of the shear stress component takes the following form (Kostas P. Soldatos, 2009):

$$\tau_{[xz]} = -\tau_{[zx]} = \frac{1}{2} m_{xy,x} = \frac{1}{2}\ d^f K_{z,x}^f = -\frac{1}{2} d^f w_{,xxx} \quad (7)$$

Where d^f is the additional elastic modulus that related to the fiber bending stiffness (Kostas P. Soldatos, 2009). It has been pointed out in (Ali. F. Farhat, 2013) and, in further related studies that for comparison reason between the solutions based on one-D and two-D methods with the exact solutions, the following notation:

$$d^f = \frac{1}{12}\ C_{11}\ l\ L \quad (8)$$

will be employed in the present paper. Such notation has to be involved to include the parameter l that of the material intrinsic length which can be considered to the fiber thickness. The full form of the shear stress presented in the case of the presence of the fiber bending stiffness is as follows (Kostas P. Soldatos, 2009):

$$\tau_{xz} = \tau_{(xz)} + \tau_{[xz]} \quad (9.a)$$

$$\tau_{zx} = \tau_{(xz)} - \tau_{[xz]} \quad (9.b)$$

Furthermore, with the use of equations (7), (8) and (9) one can write the shear stress in the following appropriate form

$$\tau_{xz} = Q_{55}\ \gamma_{xz} - \frac{1}{24}\ C_{11}\ l\ L w_{,xxx} \quad (10.a)$$

$$\tau_{xz} = Q_{55}\ \gamma_{xz} + \frac{1}{24}\ C_{11}\ l\ L w_{,xxx} \quad (10.b)$$

Consequently, the following form of the force and moment resultant can be employed in the present study:

$$N_x^c = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x\, dz,\quad M_x^a = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x \frac{z}{2}\left[\frac{h^2}{4} - \frac{z^2}{3}\right] dz,\quad Q_x^a = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{(xz)} \left[\frac{h^2}{8} - \frac{z^2}{2}\right] dz,\ \ M_x^c = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x z\, dz,\quad M_x^f = \frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}} m_{xy} dz = \frac{-C_{11} lL}{24}\int_{-\frac{h}{2}}^{\frac{h}{2}} w_{,xx} dz = \frac{-C_{11} lLh}{24} w_{,xx} \quad (11)$$

These forms will serve to formulate the governing equations of the considered present problem in the fore coming section.

5. Governing equations and boundary conditions

In order to determine the two degrees of freedom involved in the displacement field (2) those are u_1 and w, one can use the relevant two-D equations of motion presented in (Kostas P. Soldatos, 2009) for writing the following one-D equations of motion:

$$N_x^c = -\rho_1 \ddot{w}_{,x} + \hat{\rho}_0^{11} \ddot{u}_1, \quad (12.a)$$

$$M_{x,xx}^c + M_{x,xx}^f = q(x) + \rho_0 \ddot{w} + \rho_2 \ddot{w}_{,xx} + \hat{\rho}_1^{11} \ddot{u}_{1,xx} \quad (12.b)$$

$$M_{x,x}^a - Q_x^a = -\hat{\rho}_1^{11} \ddot{w}_{,x} + \hat{\rho}_0^{12} \ddot{u}_1 \quad (12.c)$$

In which will be reduced in the present paper that focus on finding the static solution of the considered problem to the following equilibrium equations:

$$M_{x,xx}^c + M_{x,xx}^f = q(x) \quad (13.a)$$

$$M_{x,x}^a - Q_x^a = 0 \quad (13.b)$$

Where the dots appear in equations (12) denote the ordinary differentiation with respect to time and the inertia terms in the right side of equations (12) will take zero value in the present case of finding the static solution which leads to the equilibrium equations (13). Moreover, the simply supported end boundary conditions at $x = 0$ and $x = L$ are as follows (Kostas P. Soldatos, 2009):

$$W = 0,\ M_x^c + M_x^f = 0\ and, M_x^a = 0 \quad (14)$$

Which will help the formulate the particular solution of the considered problem that associated with the simply supported end boundary conditions in the next section.

6. The solution of governing equilibrium equations

With the use of the displacement field (2) in connection with (6-10) on can write the force and moment resultant (11):

$M_x^c = D_{11}(-w_{,xx}) + D_{11}^a(u_{1,x}),$

$M_x^a = D_{11}^a(-w_{,xx}) + D_{11}^{aa}(u_{1,x}),$

$M_x^f = \frac{-C_{11}lLh}{24} w_{,xx}\ ,$

$Q_x^a = A_{55}\ (u_1),$

Where:

$D_{11} = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11} z^2\, dz, \quad , D_{11}^a = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{55}\ \frac{z^2}{2}\left[\frac{h^2}{4} - \frac{z^2}{3}\right] dz, ,$

$D_{11}^{aa} = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{55}\ \frac{z^2}{4}\left[\frac{h^2}{4} - \frac{z^2}{3}\right]^2 dz,$ and $A_{55} = Q_{55}\left[\frac{h^5}{320} - 4\ h\right].$ (15)

Consequently, the equilibrium equations (13) can be rewritten as a set of two simultaneous differential equations in two unknowns. These differential equations can be obtained with the use of equations (15) in terms of the displacement components yielding the following Navier-type differential equations system:

$$-D_{11}^f w_{,xxxx} + D_{11}^a u_{1,xxx} = q(x) \qquad (16.a)$$

$$-D_{11}^a w_{,xxx} + D_{11}^{aa} u_{1,xx} - A_{55} u_1 = 0 \qquad (16.b)$$

Where the rigidity D_{11}^f is related to fiber bending modulus and the rigidity D_{11} and defined as presented in (Ali. F. Farhat, 2013):

$$D_{11}^f = \frac{1}{2} h d^f + D_{11} = \frac{h}{24} C_{11} lL d^f + D_{11} \quad (17)$$

It can be seen that the number of end boundary conditions (14) meets the sixth order of the set of ordinary differential equations that with respect to the x- coordinate. The sought solution for equilibrium equations (16) is of the form that the following displacement components:

$$u_1 = A\cos(p_m x)\,, w = B sin(p_m x) \qquad (18)$$

satisfy exactly the conditions (14) at the beam ends $x = 0\ and\ x = L$. Consequently, inserting the assumption (18) into Navier-type differential equations system (16) will convert it to the following two simultaneous linear algebraic equations system of two unknown constants *A* and *B*:

$$\begin{bmatrix} p_m^4 D_{11}^f & p_m^3 D_{11}^a \\ p_m^3 D_{11}^a & p_m^2 D_{11}^{aa} - A_{55}^a \end{bmatrix} \begin{bmatrix} B \\ A \end{bmatrix} = \begin{bmatrix} q_m \\ 0 \end{bmatrix} \qquad (19)$$

In which can be solved to obtain the values of A and B. Consequently, the substitution of these obtained values in equations (18) gives the desired solution. Furthermore, based on the obtained solution, numerical results and related discussion going to be presented in the following section.

7. Results and Discussion

In a similar manner which followed by several researchers that appears in the literature to test the reliability of the considered 1D model, numerical results in the case of simply supported boundary conditions have been presented. In order to present appropriate comparisons between the results based on the obtained solution and (Ali F. Farhat & Soldatos, 15 Apr 2015), the same notation of the fibre bending stiffness elastic modulus d^f and non-dimensional parameter λ has been employed. The elastic properties of the transversely isotropic beam material are assumed as follows (Ali F. Farhat & Soldatos, 15 Apr 2015):

$E_L/E_T = 40,\ G_{LT}/E_T = 0.5,\ \ G_{TT}/E_T = 0.2,\ v_{LT} = v_{TT} = 0.25.$

Where L and T denote properties which associated with the longitudinal and transverse fibre direction, respectively.

Table 1 shows numerical results of through-thickness in-plane displacement distributions of a homogeneous beam that has thickness to length ratio of (h/L= 0.25). It can be seen that the results of perfectly flexible fibres where $\lambda = 0$ of the present solution and their counterparts of the Pagano solution (N.J. Pagano, 1969) are close to each other. Moreover, one can see that the numerical results for different values of the non-dimensional fibre bending stiffness parameter λ based on the present solution and those based on the exact asymmetric solution presented in (Ali F. Farhat & Soldatos, 15 Apr 2015) are close to each other.

Table 2 depicts numerical results for the through-thickness deflection distributions of a homogeneous beam that has thickness to length ratio of (h/L= 0.25). The deflection is computed at the medal length of the beam which is expected to be the maximum of it.

Table 1 Through-thickness in-plane displacement distributions of a homogeneous beam (h/L= 0.25)

z/h	$E_T U(0,z)/Lq_1$ at $\lambda = 0$		$E_T U(0,z)/Lq_1$ at $\lambda = .002$		$E_T U(0,z)/Lq_1$ at $\lambda = .004$		$E_T U(0,z)/Lq_1$ at $\lambda = .006$		$E_T U(0,z)/Lq_1$ at $\lambda = .008$	
	Exact	*Present*	*Exact*	*Present*	*Exact*	*Present*	*Exact*	*Present*	*Exact*	*Present*
0.5	0.129524	0.137407	0.0768314	0.039865	0.045604	0.038799	0.024773	0.037789	0.009768	0.036828
0.4	0.061373	0.066636	0.035698	0.059345	0.020603	0.057758	0.010630	0.056253	0.003527	0.054824
0.3	0.027746	0.024724	0.015502	0.056641	0.008408	0.055126	0.003805	0.053690	0.0005967	0.052326
0.2	0.011110	0.004458	0.005604	0.042561	0.002507	0.041423	0.000573	0.040344	-0.000711	0.093319
0.1	0.002564	0.001378	0.000611	0.022565	-0.000397	0.021962	-0.000949	0.021389	-0.001250	0.020846
0	-0.002681	0	-0.002369	0	-0.002063	0	-0.001763	0	-0.001470	0
-0.1	-0.007731	-0.001378	-0.005195	-0.022565	-0.003616	-0.021962	-0.002504	-0.021389	-0.001655	-0.020846
-0.2	-0.015588	-0.004458	-0.009646	-0.042561	-0.006120	-0.041423	-0.003767	-0.040344	-0.002070	-0.093319
-0.3	-0.030677	-0.024725	-0.018322	-0.056641	-0.011118	-0.055126	-0.006406	-0.053690	-0.003090	-0.052326
-0.4	-0.061091	-0.066636	-0.035972	-0.059345	-0.021421	-0.057758	-0.011980	-0.056253	-0.005398	-0.054824
-0.5	-0.122702	-0.137407	-0.071907	-0.039865	-0.042543	-0.038799	-0.023543	-0.037789	-0.010337	-0.036828

Table 2 Through-thickness deflection distributions of a homogeneous beam (h/L= 0.25)

z/h	$E_T W(\frac{L}{2},z)/Lq_1$ at $\lambda = 0$		$E_T W(\frac{L}{2},z)/Lq_1$ at $\lambda = .002$		$E_T W(\frac{L}{2},z)/Lq_1$ at $\lambda = .004$		$E_T W(\frac{L}{2},z)/Lq_1$ at $\lambda = .006$		$E_T W(\frac{L}{2},z)/Lq_1$ at $\lambda = .008$	
	Exact	*Present*	*Exact*	*Present*	*Exact*	*Present*	*Exact*	*Present*	*Exact*	*Present*
0.5	-1.209112	-1.115430	-0.911522	-0.674378	-0.737394	-0.656338	-0.623020	-0.639238	-0.542095	-0.623007
0.4	-1.188321	-1.115430	-0.890188	-0.674378	-0.715752	-0.656338	-0.601183	-0.639238	-0.520126	-0.623007
0.3	-1.167960	-1.115430	-0.870126	-0.674378	-0.695899	-0.656338	-0.581495	-0.639238	-0.500578	-0.623007
0.2	-1.149490	-1.115430	-0.852186	-0.674378	-0.678317	-0.656338	-0.564187	-0.639238	-0.483496	-0.623007
0.1	-1.133595	-1.115430	-0.836755	-0.674378	-0.663210	-0.656338	-0.549336	-0.639238	-0.468861	-0.623007
0	-1.120566	-1.115430	-0.823983	-0.674378	-0.650641	-0.656338	-0.536944	-0.639238	-0.456629	-0.623007
-0.1	-1.110468	-1.115430	-0.813889	-0.674378	-0.640600	-0.656338	-0.526978	-0.639238	-0.446749	-0.623007
-0.2	-1.103195	-1.115430	-0.806396	-0.674378	-0.633022	-0.656338	-0.519378	-0.639238	-0.439163	-0.623007

-0.3	-1.098430	-1.115430	-0.801305	-0.674378	-0.627773	-0.656338	-0.514053	-0.639238	-0.433807	-0.623007
-0.4	-1.095495	-1.115430	-0.798210	-0.674378	-0.624605	-0.656338	-0.510853	-0.639238	-0.430597	-0.623007
-0.5	-1.093010	-1.115430	-0.796296	-0.674378	-0.623027	-0.656338	-0.509498	-0.639238	-0.429402	-0.623007

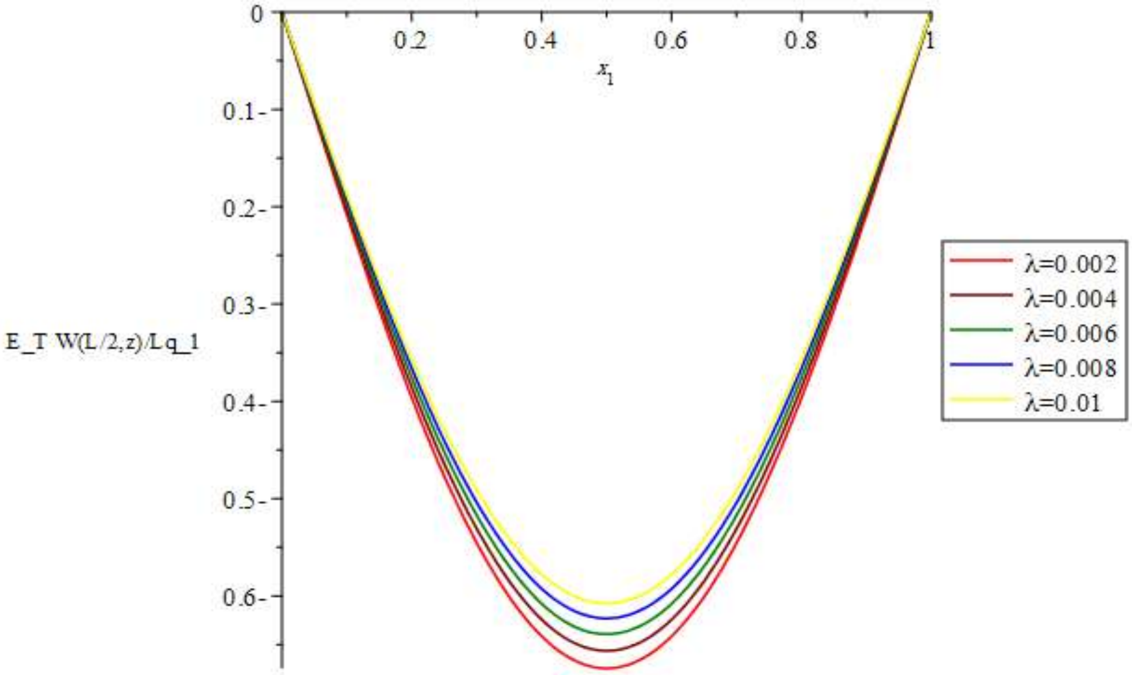

Figure 1 the deflection distributions of a homogeneous beam (h/L= 0.25) at different values of Lamda

It can be seen that the deflection based on the present solution does not change through the beam thickness which is caused by the independent displacement function w on z. In addition, the deflection at the different values of the non-dimensional fibre bending stiffness parameter λ is close to the average of it obtained on the exact solution. Furthermore, it is clear as expected that as the value of the parameter λ increases, the values of the deflection decreases.

The deflection distributions along the length of a homogeneous considered beam with thickness to length ratio of (h/L= 0.25) at different values of λ is shown in the figure 1. One can see obviously, again, that as the value of the parameter λ increases, the values of the deflection decreases.

8. Conclusion

Based on the presented numerical results and the related discussion, it can be concluded that although the ratio h/L= 0.25 which characterises a very thick beam, the presented results of displacement functions U and W based on the present solution are close to those based on the exact asymmetric solution. Furthermore, the rigidity of the beam caused by the fibres bending stiffness increase as the values of the parameter λ increase when the values of the deflection decrease. Finally, it can be recommended that solutions for the case of different boundary conditions can be obtained with the use of the presented theory with the consideration of the fibres bending stiffness.

Appendix

In the appendix, the following notation is going to be provided:

Table of some used notation.

The sample	What it is stand for.
$U(x,z,t)$ and $W(x,z,t)$	The displacement functions along x and z directions respectively.
ε_x	Normal strain.
γ_{xz}	Shear strain.
σ_x	Normal stress.
τ_{xz}	Shear stress.
$\tau_{(xz)}$	The symmetric part of the shear stress component.
$\tau_{[xz]}$	The anti-symmetric part of the shear stress component.
m_{xy}	The couple-stress tensor.
$N_x^c, M_x^a, Q_x^a, M_x^c, M_x^f$	The force and moment resultant.
$C_{11}, Q_{11}, Q_{55}, E_L, E_T, G_{LT}, G_{TT}, v_{LT}, v_{TT}$	The elastic properties of the beam material.
d^f	The fibre bending stiffness elastic modulus.
λ	the non-dimensional fibre bending stiffness parameter λ
l	The material intrinsic length parameter

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تطبيق جديد لنظرية الدعامة المرنة لامبارتسميان

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الملخص

تطبيق جديد لنظرية الدعامة (القضبان) المرنة التي لها درجتي حرية والتي تأخذ بعين الاعتبار قابلية الالياف لمقاومة الثني هو الهدف الرئيسي للدراسة المعروضة بهذا البحث. في هذه النظرية المطبقة بهذا البحث كانت دالة التشكيل لامبارتسميان (1958) متضمنة في فرضية دوال الازاحة المستخدمة. باستخدام هذه الفرضية لدوال الازاحة تم إيجاد الانفعال و الاجهاد المتضمن الجزء غير المتماثل لإجهاد القص. بالإضافة الى ذلك تم إيجاد حل معادلات الاتزان الضابطة للمسألة موضع الدراسة والمتضمن حدود ذات العلاقة بمبدأ صلابة الالياف المقاومة للثني. تم إيجاد وعرض نتائج عددية للحل المتحصل عليه مرفق بتحليل ومناقشة لهذه النتائج المتحصل عليها.

الكلمات المفتاحية: دالة التشكيل، صلابة الالياف المقاومة للثني، الجزء غير المتماثل لإجهاد القص.